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Universal scaling behaviour in weighted trade networks

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Abstract. Identifying universal patterns in complex economic systems can reveal the dynamics and organizing principles underlying the process of system evolution. We investigate the scaling behaviours that have emerged in the international trade system by describing them as a series of evolving weighted trade networks. The *maximum-flow spanning trees* (constructed by maximizing the total weight of the edges) of these networks exhibit two universal scaling exponents: (1) topological scaling exponent $\eta = 1.30$ and (2) flow scaling exponent $\zeta = 1.03$.

PACS. 89.75.Da Systems obeying scaling laws – 89.75.Hc Networks and genealogical trees – 89.65.Gh Economics; econophysics, financial markets, business and management

1 Introduction

In recent years, some physicists have focused their research on complex economic systems comprising multiple agents that adapt or react to the pattern that is collectively created by these agents [1]. Investigating the regularities or universal patterns in the evolution of real-life complex economic systems can help to reveal the potential evolution mechanisms and design principles of these systems. Fortunately, successful empirical results have been obtained [2], including (1) path dependence or nonergodicity of economic history illustrated by the famous example of QWERTY keyboard configuration [3], (2) Zipf's law related to the frequency of objects and their sizes, for example, firm size distribution [4], and (3) power-law fluctuation behaviours in the growth of firms and GDP [5,6]. With complex networks emerging as a hot research field, the interaction patterns among agents have attracted researchers' interests. It is found that many economic networks display the typical characteristics of complex networks, such as scale-free degree distribution, the small-world property, a high clustering coefficient, and degree–degree correlation behaviours [7–9]. Although all the previous results have shed some light on understanding the economic complexity, hitherto, to the best of our knowledge, no universal scaling behaviours characterizing the interaction strength patterns have been discovered in the evolution process of complex economic systems. Furthermore, according to a recent study, investigating the evolving patterns among sequent networks can deepen our understanding of complex networks [10]. In this paper, we describe the international trade system as a series of weighted trade networks in which nodes represent countries; edges, trade relationships existing in every pair of nodes; and weights, the volume of trade flow between two nodes. By investigating the transport property of the allometric scaling [11,12] of trade networks, we find two scaling exponents of a topological scaling exponent $\eta = 1.30$ and a flow scaling exponent $\zeta = 1.03$. Although trade institutions, trade environment, and individual trade patterns have changed significantly, the two exponents are universal. Our study extends the spanning tree method in [12] by (1) maximizing the interaction strength in the tree (2) introducing the flow scaling exponent to characterize the spanning tree, and (3) providing a new viewpoint to investigate the simultaneous evolution of topology and weight for weighted complex networks.

2 Data

The data analysed in this paper are obtained from the 'Expanded Trade and GDP data' version 4.1, which provides estimates of trade flows between independent states (1948–2000) and GDP per capita of independent states (1950–2000) [13]. In any given year, each dyad AB arising from two countries A and B, in principle, has the following four trade flows: (1) exports from A to B, (2) imports by A from B, (3) exports from B to A, and (4) imports by B from A. In principle, exports from A to B should equal imports by B from A, and vice versa. However, these accounting identities are often not observed recorded trade data. Furthermore, trade records are valued in different formalisms. Exports are often valued as f.o.b. ('free on board'); however, in some cases, they are valued as c.i.f. ('customs and freight included'). Similarly, only partial imports are valued as c.i.f. and others are valued as

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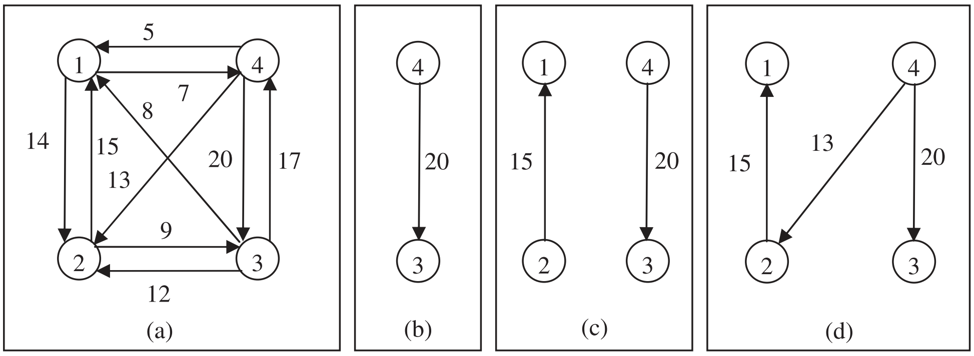

Fig. 1. Construction procedure of the MFST of the weighted trade network. (a) Example of a weighted trade network with four nodes. Each number near the arrows is the value of trade flow between every pair of nodes. (b) Selecting the largest weighted edge as the first edge of the MFST. (c) Adding an edge to the MFST according to the priority of the edge weight. (d) In the final MFST, the root node is '④'.

f.o.b. In order to avoid confusion, we have adjusted the original data based on two principles: (1) all exports are valued as f.o.b. and all imports are valued as c.i.f. and (2) we construct two trade networks for each year by using all import records or all export records separately. Evidently, the two networks have the same network structure and only differ in terms of including/excluding freight fee in each trade flow. We have analysed the two classes of networks and have obtained the same results. Therefore, we only refer to the networks constructed by all export records in the latter. We describe the international trade system as a series of evolving weighted trade networks (w_{ij}^t) [14], where node i represents countries; directional edge (i, j), the trade relationship existing from node i to j; weight w_{ij}, the volume of trade flow between (i, j); and $t = 1$ represents the year 1948. The data spans from 1948 to 2000; therefore, there are a total of 53 weighted trade networks.

3 Method

We consider the weighted trade networks as transport networks whose function is to deliver resources from every source country to every other country in the global trade network. This is possible because the directionality of import or export relationship defines a 'flow' of goods, services, capital, and information between the nodes of these networks [7]. For each node, its import edges carry incoming flows that denote the paths to import goods from other nodes, and its export edges carry outgoing flows that denote the paths to export goods from other nodes. Compared to food webs [12], in which energy flows from inorganic matter to plants and from plants to higher trophic levels form a single outgoing flow direction, goods and energy flows transferred along import edges and export edges separately form an incoming flow direction and an outgoing flow direction. Although there is a difference between trade networks and food webs, they are similar in terms of exchanging materials and energies among their nodes, and their skeleton networks (i.e., chain-length minimization spanning tree in food webs and interaction-strength maximization spanning tree in trade networks) carry out most energy or goods flows. In order to apply the spanning tree approach to study the statistical property of the skeleton of the trade networks, we replaced the chain-length minimization criterion by the interaction-strength (trade flow in links) maximization criterion to identify the main transport pathways [15]. We define the *maximum-flow spanning tree* (hereafter referred to as MFST) of a weighted trade network as the spanning tree with a set of edges that maximize the summation of their edge weight. The MFST represents the main transport pathways in each trade network. Similar to the minimum spanning tree algorithm [16], the construction procedure was implemented as follows. We select an import edge according to the priority of the edge weight and add it to the tree if it does not make any loop and if the number of the import edges of each node is not increased to more than 1. We repeat the previous step until the tree includes all the nodes (see Fig. 1). In the final MFST, there must exist only one node termed as the root node that has only export edges and no import edges. This characteristic emerges from the computational procedure of the MFST to avoid generating a loop between nodes. This does not mean that the root node should not buy goods to survive; instead, it may mean that the root node plays an important role in the trade networks. In real trade data, the root node often corresponds to the USA. The selected edges in the MFST represent 'strong' links and the residual edges represent 'weak' links that shorten the paths on the spanning tree.

In order to validate the importance of the MFST, we compute the *link ratio* — the ratio of the number of edges in the tree and that of the network — and the *flow ratio* — the fraction f of the edge weight summation over the selected edges and that over the total edge. As shown in Figure 2, 20%∼35% of the total trade flow has been transferred to the MFSTs in which the number of links is less than 3% (even 1%) of the total number of links in the whole network. We can conclude that the MFST represents the main trade relationship of the whole network. Similar spanning trees are also investigated, for example,

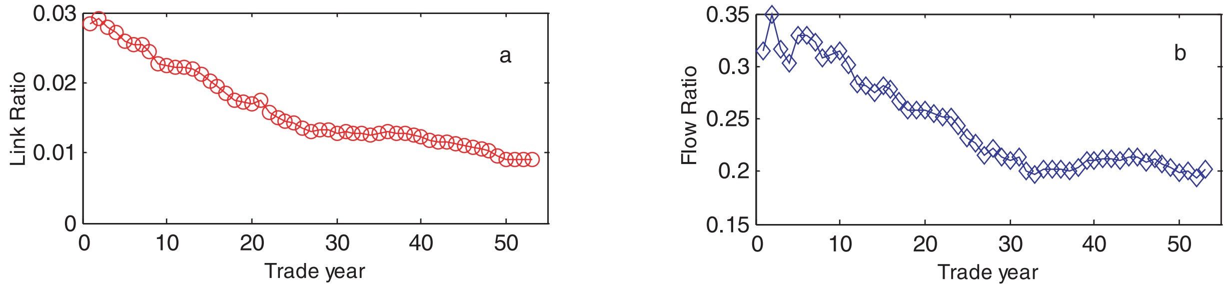

Fig. 2. Link ratio is defined as the number of links in the MFST divided by the total links in the whole trade network, and flow ratio is defined as the volume of trade flow in the MFST divided by the total volume of trade flow in the whole trade network. Each data point corresponds to every weighted trade network and its MFST.

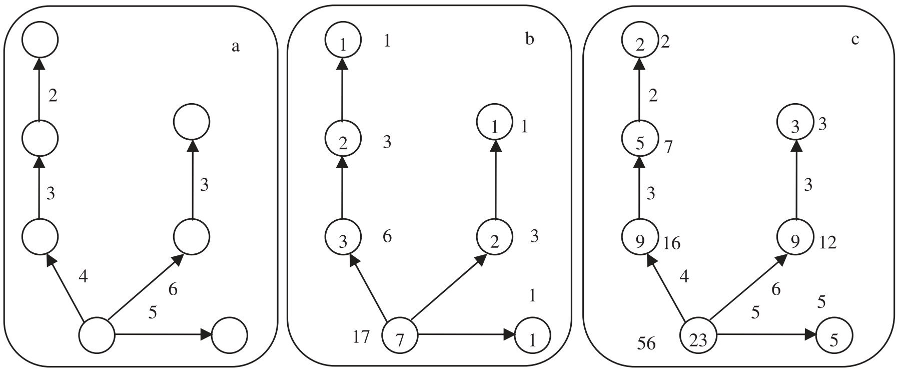

Fig. 3. Illustration of computing A_i, C_i, fA_i, and fC_i from an MFST of the trade network. (a) Example of an MFST. (b) Computing A_i (at the centre of each node) and C_i (outside the circles) for each country i. (c) Computing fA_i (at the centre of each node) and fC_i (outside the circles) for each country i. In (a) and (c), the data near each arrow line represent the volume of trade flow.

the spanning tree constructed by maximizing the total edge between centralities [17] and the 'high-flux backbone' of metabolism in the metabolic network [18].

4 Results

After constructing the MFST using the whole network, we can now study the manner in which materials, energies, and information are exchanged in the international trade system. We analysed transport properties in two forms of network structures and trade flows. For topology, for each country i in the MFST, we compute A_i of countries trading directly or indirectly with i along with country i itself, and $C_i = \sum_k A_k$, where k runs over the set of exporting countries of i along with i itself (see Fig. 3b). Note that $i = 0$ denotes the root node. In a transport system, A_i is proportional to the quantity of resources (goods or services) exchanged at point i, and C_i measures the 'cost' of this transfer. Evidently, both A_i and C_i ignore the information contained in the strength of interactions. In order to remedy this deficiency, we compute fA_i of the import trade flow along with the accumulated export trade flow of node i, and $fC_i = \sum_k fA_k$, where k runs over the set of exporting countries of i plus i itself (see Fig. 3c). Similarly, fA_i and fC_i represent the weighted quantity of resources and the weighted transferring cost, respectively.

For each weighted trade network, we first computed C_i and A_i and then measured C_i as a function of A_i. Averaged over all countries, we obtain the scaling relation $C \propto A^\eta$, where η named as the *topological scaling exponent* characterizes the structure of the MFSTs. Figure 4 shows the results of ten sample trade networks, in which the slope of the solid line is the scaling exponent determined by the best power-law fit. Surprisingly, all values fluctuate in the range of 1.30 ± 0.03.

Now, we analyse similar statistical properties of the trade flow for each MFST. After computing fC_i and fA_i for each country, we measure fC_i as a function of fA_i and find the scaling relation $fC \propto (fA)^\zeta$ by averaging it over all countries. The *flow scaling exponent* ζ characterizes the trade flow pattern of the MFSTs. Figure 5 shows the results of the ten sample trade networks in which the slope of the solid line is the scaling exponent determined by the best power-law fit. All values are marginally consistent with 1.03.

In order to validate the two scaling exponents are fundamental properties of the international trade system rather than properties of the methodology used [19], we analysed 1060 (20 for each of the 53 weighted trade networks) randomly constructed spanning trees with real-life trade data. About 70% of the random spanning trees do not exhibit any scaling behaviours. Some random spanning trees exhibit scaling behaviours, but their scaling exponents are scattered and different from the value observed in the MFST. For example, Figure 6 illustrates the difference between a random spanning tree and an MFST by plotting A, C, fA, and fC for the MFST and two random spanning trees, all of which are constructed by the weighted trade network in 1990.

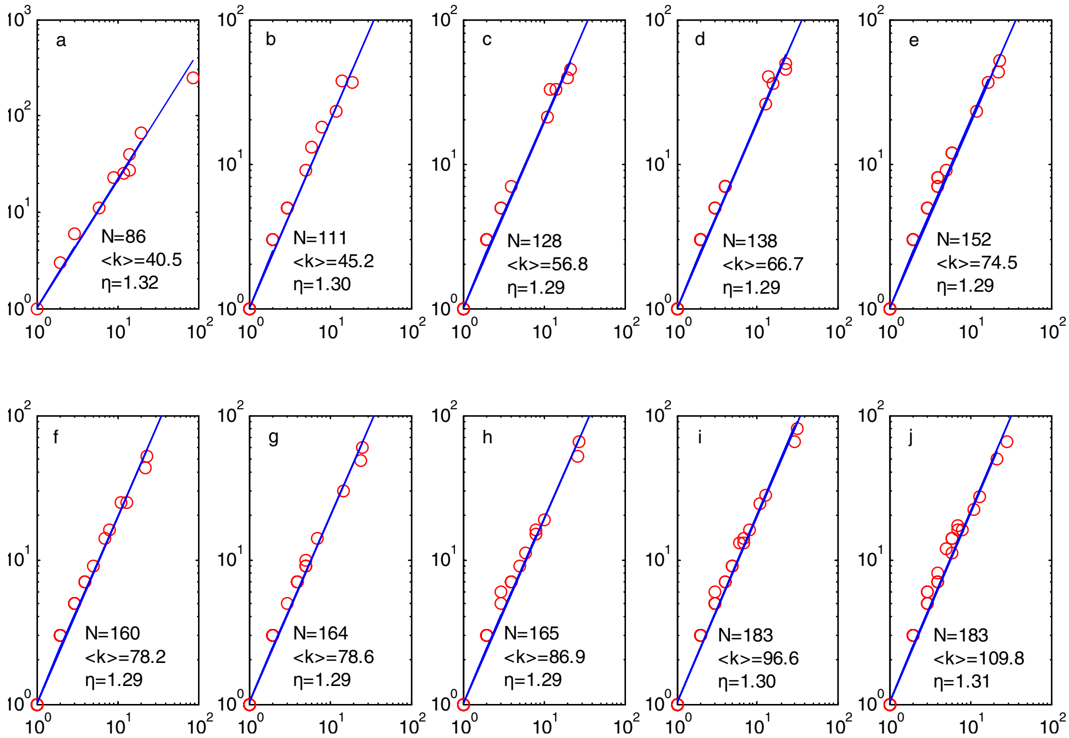

Fig. 4. Scaling relations of C against A. Plots of C_i (vertical axis) versus A_i (horizontal axis) in double logarithmic scales and the best power-law fit to the data. Subfigures (a)–(j) correspond to the data of trade years 1955, 1960, 1965, 1970, 1975, 1980, 1985, 1990, 1995, and 2000, respectively. The number of countries N has increased more than two times from 86 in 1955 to 183 in 2000. The average number of trade partners $\langle k \rangle$ has also increased 2.5 times from 40.5 to 109.8. The topological scaling exponent η varies from 1.29 to 1.32 and its average value based on all the data is 1.30.

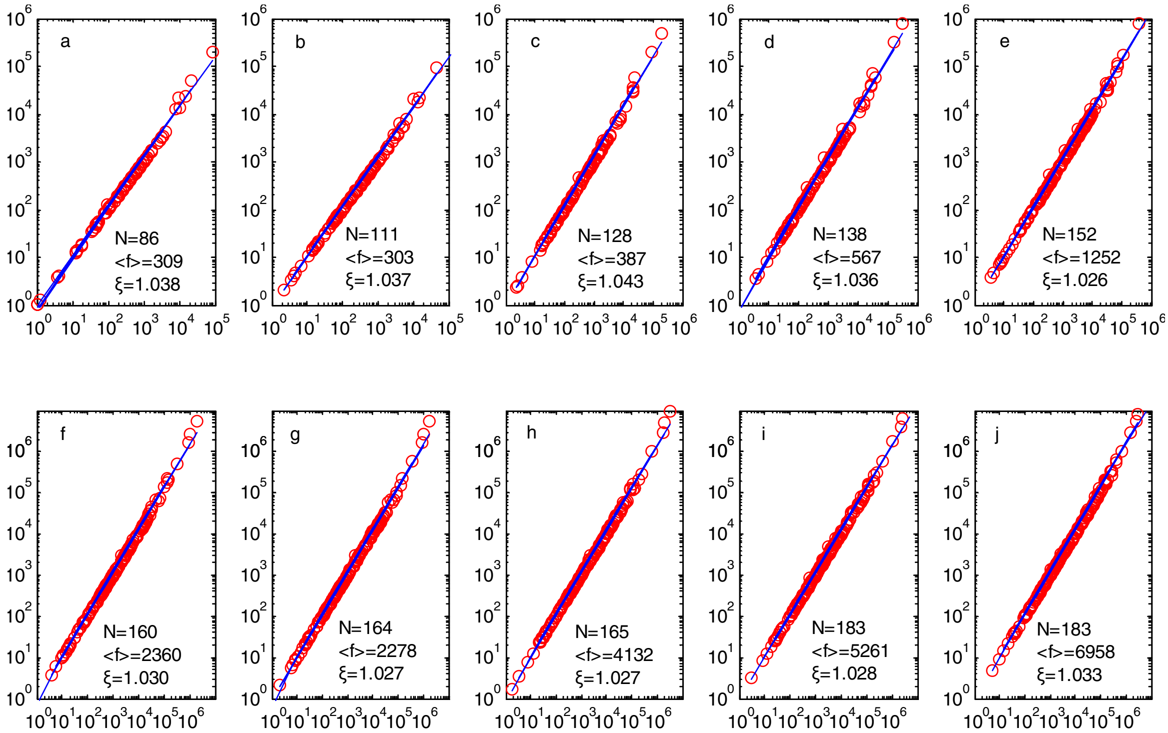

Fig. 5. Scaling relations of fC against fA. Plots of fC_i (vertical axis) versus fA_i (horizontal axis) in double logarithmic scales and the best power-law fit to the data. Subfigures (a)–(j) correspond to the trade years 1955, 1960, 1965, 1970, 1975, 1980, 1985, 1990, 1995, and 2000, respectively. The average volume of trade flow of each link in the MFSTs $\langle f \rangle$ increased nearly 23 times from 309 in 1955 to 6958 in 2000. Flow scaling exponent ζ varies from 1.026 to 1.043, and its average value based on all data is 1.03.

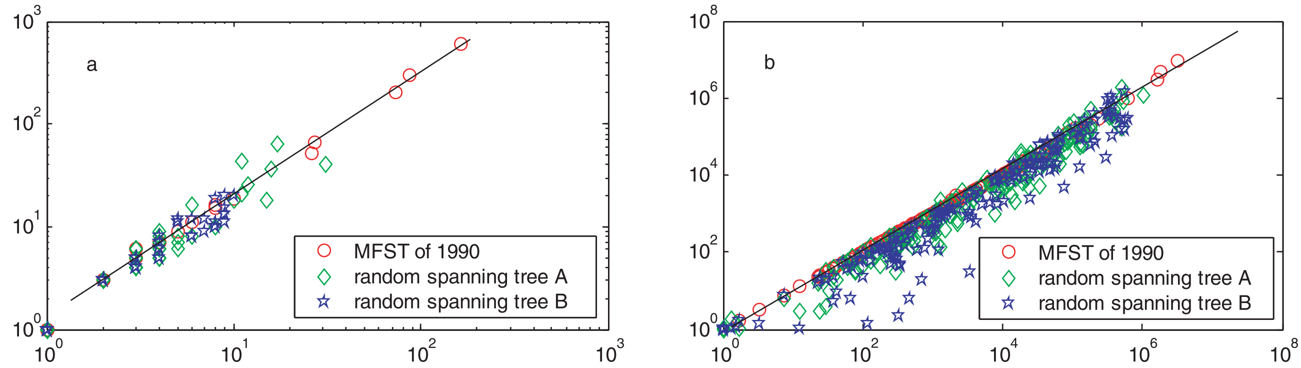

Fig. 6. Comparing the scaling behaviour of the MFST and two random spanning trees that are constructed by the weighted trade network in 1990. (a) Plots of C_i (vertical axis) versus A_i (horizontal axis) in double logarithmic scales. (b) Plots of fC_i (vertical axis) versus fA_i (horizontal axis) in double logarithmic scales. The two lines in (a) and (b) are the best power-law fit to the topological and flow scaling behaviour of the MFST.

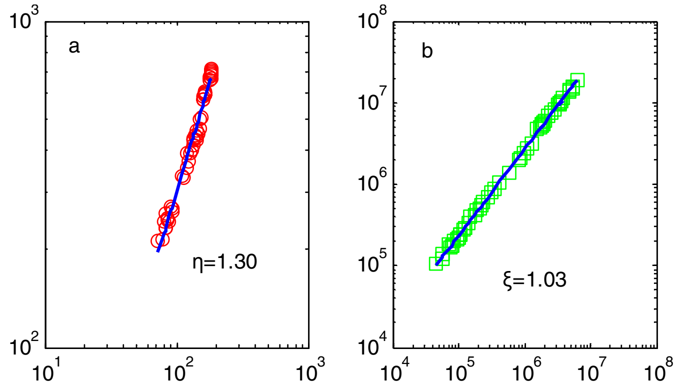

Fig. 7. Scaling relations of C against A and fC against fA. Plots of C_0 (vertical axis) versus A_0 (horizontal axis) and fC_0 (vertical axis) versus fA_0 (horizontal axis) in double logarithmic scales for the data of 53 weighted trade networks (note that the scales in the two subfigures are different). Best power-law fit of topological scaling exponent $\eta = 1.30 \pm 0.02$ and flow scaling exponent $\zeta = 1.03 \pm 0.01$.

Combining all the results of topological scaling exponent η and flow scaling exponent ζ, we find that the two scaling exponents are independent of other topological and flow quantities. In the analysed trade data, the size of the trade network or the number of countries is 73 for the year 1948 and 183 for the year 2000, which indicates an increase over 2.5 times. Furthermore, the average node degree or the number of trade partners also varies to the extent that the least value is 34.7 and the highest value is 109.8. In particular, the average trade flow of each link in the whole trade network and in the spanning tree has increased over 22 and 35 times, respectively, in the last fifty years. Remarkably, all networks have two scaling exponents of η and ζ that are roughly consistent with 1.30 and 1.03, respectively. Averaged over all networks, we believe that the mean (expected) values of $\eta = 1.30$ and $\zeta = 1.03$ may represent the appropriate universal behaviours underlying each network. Minor data errors or statistical deviations might incur the two exponents violating the ideal values 1.30 and 1.03. Considering that the root node is less disturbed by perturbations, we select only the behaviours of root node for each trade network and plot the data of C_0 against A_0 and fC_0 against fA_0 for all trade networks. As shown in Figure 7, surprisingly, we again find two scaling relations fitted by topological scaling exponent $\eta = 1.30 \pm 0.02$ and flow scaling exponent $\zeta = 1.03 \pm 0.01$, which indicate that these scaling behaviours might indeed be invariant and universal in the international trade system.

5 Discussion

By analogy with the ecosystems, we show that the international trade system maintains a similar balance between adaptation and adaptability [20] by relating the 'strong' and 'weak' links to the adaptation and adaptability properties, respectively. In the last fifty years, we have observed great changes in the global trade system in terms of at least three aspects. Firstly, trade institutions have evolved from a bilateral trade framework into a multiple or global trade framework due to the establishment of the World Trade Organization in 1995. Secondly, the most significant changes in trade environments have occurred as the result of two extraordinary political events — the cold war in the 1950s and the recession in the 1990s. Lastly, owing to the substantial movement towards trade liberalization during the post-war period, the trade patterns of individual countries have experienced statistically significant changes in their export-GDP and import-GDP ratios [21]. Specifically, most trade ratios exhibited a structural break in their time paths in the 1970s or 1980s, and the post-break trade exceeded the pre-break trade in the case of many countries. Although all these changes have greatly influenced the interaction pattern and the evolution of the international trade system [22,23], our results suggest that these evolutionary constraints do not affect the shape and flow patterns of the spanning trees, which seem to be invariant and universal. The 'strong' and 'weak' links might have complementary roles: the former determine the adaptation properties of optimizing the exchange efficiency of the trade networks, while the latter form the local loops in a manner that increases adaptability in order to retain the robustness of the trade networks undergoing substantial and drastic changes [24]. We are not clear about the mechanisms that make the global trade system adapt to these institutional, economic, political, and technological changes; however, we believe that there must exist a single, common organizing principle under the entire evolution process of the global trade system in order to optimize the exchange efficiency of the global trade system.

6 Conclusion

In conclusion, we have identified two universal scaling exponents to characterize the topological structure and trade flow pattern of the maximum-flow spanning trees. Considering that the radical changes in trade institutions, trade environments, and individual trade patterns do not affect the structure and trade flow pattern of the spanning trees, there must exist a fundamental mechanism that governs the evolution of the international trade system to maintain a balance between adaptability and adaptation.

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